1. **Probability that the sum of two rolls of a fair 6-sided dice is less or equal to 3?**

Outcomes of 11 12 21 gives this result. Total of outcomes. So

1. **Toss a fair coin until we see 4 Heads OR 4 Tails. Probability that we stop after exactly 5 tosses?**

Outcomes are H + 4T or 4H + T. Outcomes of an isolated T or H in first 4 tosses give this result. Total of outcomes. So

1. **A and B flip a fair coin in turns. The first person who flips a H wins $30 from C.** 
   1. **What is the EV of the game for A when the person going first is random?**

By symmetry this is equal to

* 1. **What the EV for the game for A when A is going first?**

Let be the probability that the second person wins. Then (why? Because second person’s probability of winning is equal to the first person's probability conditional on the first person's not flipping a head with ½ chance). This gives us of EV for the first person with a $30 payout.

* 1. **Assume now that the winner of the game takes $30 from the loser, instead of taking it from C. What is the maximum that A is willing to pay B to go first?**

Let be the max payment that A is willing to offer in $ terms. Then

* 1. **Assume now that C decides who should go first. What is the maximum that A is willing to pay C to go first?**

This amount in $ terms is equal to the difference of the EV between going first and going second. It is given by

* 1. **Why is the answer in part c and d different?**

This is a more discussion-based question. But the idea is that if A and B transfer the power of deciding who to go first from between themselves to a third entity, they have lost something of value.

**6) On average a train comes to the station every 10 minutes. There is a non-zero probability that it does not arrive on time. We have just arrived at the station and know nothing about when the last train left. Is there a greater than ½ probability that our waiting time is more than 5 minutes?**

There is a non-zero chance that some wait intervals are longer than others. The probability that we arrive on longer intervals is greater than arriving in shorter intervals. Given that we have arrived in longer intervals, the wait time is likely to be longer than 5 minutes. The answer is . Note that the expected wait time is still exactly 5 minutes. The idea is that the distribution of waiting time is not symmetric.

**7) We interview teachers and students about the size of their classes and take the mean of each group’s responses. Which group will have a higher mean?**

will have a higher mean as a group. Why? Consider the extreme example of there only being 2 classes. One with 1 teacher and 200 students, and one with 1 teacher and 10 students. Apply the same principle now to less extreme examples.

**8) There are 50 ropes in a box. We randomly choose the ends of ropes in the box and connect them together. What is the expected number of loops that is formed when we have tied all loose ends together?**

The first time we do this there will be exactly 49 loose ropes left (either a loop created or two loose ropes “merge”) and in expectancy we would have created 1/99 loops since there are 100 loose ends. The second time we do this there are 48 ropes left, and in expectance we would have created 1/97 loops. Use linearity of EV.

**100 ants are put randomly on a 1-dimentional line segment of 100 units in length at random orientations. They walk till they fall off and if they bump into each other, they swap directions. If they walk at a speed of 1 unit per second, what is the longest time it will take for all the ants to fall off?**

The ants are fungible, and we can imagine that at collision, if we swap their identities, then it is as if a collision never occurred. Therefore, the maximum amount of time it takes is exact .

**10) There is 1 sheep and 100 tigers on an island. The rules are as follows: the tigers want to eat the sheep, but upon consumption the tiger turns into a sheep, and the tigers do not want to be eaten. Should any of tigers eat the sheep?**

Case of 1 tiger eat. Case of 2 tigers do not eat. Case of 3 tigers eat, reduction to case of 2. By induction eat on odd # and do not eat on even #. So